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Reg. No.....

Name.....

M.Sc. DEGREE (C.S.S.) EXAMINATION, APRIL 2019

Fourth Semester

Faculty of Science

Branch I (A) : Mathematics

MT 04 E13—ALGORITHMIC GRAPH THEORY

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Construct a graph of order 5 whose vertices have degree 1, 2, 2, 3, 4. What is the size of this graph ?
2. Define an algorithm and complexity of an algorithm.
3. Explain the minimum spanning tree problem.
4. Define radius and diameter of a graph G.
5. Show that if G is an n-connected graph, then $G + K_1$ is $(n + 1)$ – connected.
6. Define edge-connectivity of a graph. Find $\lambda(k_{m,n})$.
7. Define perfect matching of a graph G.
8. State Kirkman's school girls problem.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Give an example of a graph G with $K(G) = \delta(G) = \Delta(G) = 2$.
10. Give a method to represent graphs in a computer.

Turn over

11. If T is a binary tree of height h and order p then prove that $h+1 \le p \le 2^{h+1}-1$.

12. Write an algorithm to find $d(u,v)$ for a fixed vertex u in a graph G and every vertex v of G.

13. Define the connection number C (G) of a graph G. Find C (k_p) and C (c_p).

14. Let G be a graph of order p and let n be an integer with $1 \le n \le p-1$. If $\delta(G) \ge \frac{p+n-2}{2}$ then prove that G is n-connected.

15. Prove that the Peterson graph is not 1-factorable.

16. In a (b, v, r, k, λ) design, prove that $\lambda(v-1) = r(k-1)$.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Prove that a non-trivial graph G is bipartite if and only if G has no odd cycle.

18. Explain the greedy algorithm and its complexity.

19. (i) Let T be a tree of order m and let G be a graph with $\delta(G) \ge m-1$. Then prove that T is isomorphic to some subgraph of G.

    (ii) Prove that a directed tree T is a rooted true if and only if T contains a vertex r with id $r = 0$ and id $v = 1$ for all other vertices v of T.

20. Explain Max-Flow Min-Cut Algorithm.

21. Explain a maximum matching algorithm for bipartite graphs and find its complexity.

22. Prove that a non-trivial graph G has a 1-factor if and only if, for every proper subset S of V (G), the number of odd components of G-S does not exceed $|S|$.

(3 × 5 = 15)